Multi-Higgs-doublet models: broad picture and hidden gems

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

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1. SM and its problems

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The Standard Model

- $u$: up quark
- $c$: charm quark
- $t$: top quark
- $d$: down quark
- $s$: strange quark
- $b$: bottom quark
- $\nu_e$: electron neutrino
- $\nu_\mu$: muon neutrino
- $\nu_\tau$: tau neutrino
- $e$: electron
- $\mu$: muon
- $\tau$: tau
- $\gamma$: photon
- $Z$: Z boson
- $W$: W boson
- $g$: gluon
left fermions

right fermions

\[ u \quad c \quad t \]
\[ d \quad s \quad b \]
\[ \nu_e \quad \nu_\mu \quad \nu_\tau \]
\[ e \quad \mu \quad \tau \]

W_1 \cdots W_3

SU(2)

U(1)

SU(3)

B

Igor Ivanov (CFTP, IST)

NHDMs

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**The Standard Model**

**Brout-Englert-Higgs mechanism** based on the Higgs doublet \( \Phi \rightarrow \) regrouping the bosons:

\[
W^+, W^-, W^3, B, H^+, H^-, H^0, h, Z, \gamma, h.
\]
The Standard Model:

- minimalistic, fully predictive theory,
- extremely efficient in describing **collider data**.

But there are several observations which the SM

- **cannot accommodate** (DM, baryon asymmetry of the Universe)
- **can describe but not explain** (fermion masses, mixing, CP violation, neutrinos).
Looking beyond the Standard Model

There must exist physics Beyond the Standard Model!

Theorists have proposed

\[ \sim 1000 \text{ models of New Physics!} \]

We just don’t know which one corresponds to reality!

The main goal of the present-day experimental HEP is to find New Physics.
Multi-Higgs-doublet models
Non-minimal Higgs sectors: a conservative approach to New Physics.
Several Higgs generations

Simple idea

Higgses can come in generations $\rightarrow N$-Higgs-doublet models (NHDMs).

- **T.D. Lee, 1973**: 2HDM as a new source of $CP$-violation (CPV);
- **Weinberg, 1976**: 3HDM with natural flavour conservation and CPV;
- Intense activity in 70–80’s: looking for a rationale behind hierarchical quark and lepton masses and mixing patterns;
- **1990–2000’s**: MSSM requires two Higgs doublets;
- In total, $\mathcal{O}(10^4)$ papers over 40 years.
Counting Higgses

- **SM**: $\phi = (\phi^+, \phi^0)$; 4 real degrees of freedom; 3 bosons become $W_L^+, W_L^-, Z_L$; 1 Higgs boson remains.
- **2HDM**: $\phi_1, \phi_2$; $2 \times 4 = 8$ real degrees of freedom.

5 Higgs bosons remain:

Exciting phenomenology at colliders, see review Branco et al, arXiv:1106.0034 ($\approx$ 1400 citations so far).
Higgs boson discovery

July 4th, 2012: Higgs searches → Higgs exploration

LHC Run 1 (20 fb$^{-1}$) + early Run 2 (80 fb$^{-1}$) statistics:
- no other Higgs bosons detected,
- very SM-like couplings of the 125 GeV Higgs.
2HDM phenomenology

This is not a blow to multi-Higgs models:

- One Higgs can be very similar to SM, the other Higgses are almost hidden
  → don’t expect democracy among Higgses!

It arises naturally in many multi-Higgs models (Higgs alignment).

- Strategy: search for hints of non-standard Higgs interactions:
  - flavour-violating decays, such as \( h \rightarrow \tau \mu \), e.g. [Aristizabal Sierra, Vicente, 1409.7690];
  - deviations in rare decays, e.g. [Modak et al, 1607.07876];
  - unusual \( hh \) production, e.g. [Ren et al, 1706.05980].

- This is the main goal of future Higgs factories: CEPC, FCC-ee, ILC/CLIC, see e.g. [An et al, 1810.09037].
In the SM the poor single Higgs doublet does all the job:

$$\Gamma_{ij} \, (\text{right quarks}_i \times \text{Higgs} \times \text{left quarks}_j), \quad i, j = 1, 2, 3$$

- masses to up-type quarks,
- masses to down-type quarks,
- mixing and CP violation for quarks.

But no resources left to explain anything...
$N$ generations of Higgses can do a lot!

$$\sum_a \Gamma^{(a)}_{ij} (\text{right quarks}_i \times \text{Higgs}_a \times \text{left quarks}_j)$$

- In general, $\Gamma^{(a)}_{ij}$ are unconstrained complex matrices $3 \times 3 \rightarrow$ too much freedom, complete mess.

- Suppose there is flavour symmetry group $G$ which acts on quarks and Higgses $\rightarrow$ each $\Gamma^{(a)}_{ij}$ can be very simple, symmetry-constrained!

- Vacuum expectation values $v_a$ break the flavour symmetry $\rightarrow$ relations among masses/mixing/CP-violation may remain.
Quark masses and mixing in NHDM

Lots of activity 70-80’s: guess $G$, guess representations, arrange for symmetry breaking $\rightarrow$ **deduce relations** among masses/mixing/CP violation.

- permutation symmetry groups $S_3$ or $S_4$: [Pakvasa, Sugawara, 1978, 1979, + Yamanaka, 1982] $\rightarrow$ perfectly (for early 80’s!) reproduced CKM;

- rephasing + permutations: $\Delta(54)$ which makes $\Gamma^{(a)}$ very simple [Segre, Weldon, Weyers, 1979]: mass hierarchy may come from $\nu_1 \ll \nu_2 \ll \nu_3$.

- typical prediction for top mass: 20–40 GeV; decline of activity in 90’s;

- **renewed interest** in last years with many strong results, see review [Ivanov, arXiv:1702.03776].
\textbf{CP violation in NHDM}

- \textbf{Spontaneous CP-violation [T.D.Lee, 1973]:}
  - lagrangian is \textit{CP}-invariant, $\Gamma$'s are real;
  - the position is minimum, $v_a$, is complex;
  - CKM becomes complex.

Recent resurrection of the idea: [Nebot, Botella, Branco, 2018].

- \textbf{Geometric CP-violation [Branco, Gerard, Grimus, 1984]:} very stable, rigid prediction for the CP-violating phase;

- Higgs exchanges as a \textit{new source of CP-violation [Weinberg, 1976];}

  - new form of \textit{CP}-symmetry (CP4 3HDM) [Ivanov, Silva, 2016] with peculiar phenomenology.
CP violation in NHDM

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Scalar dark matter

Inert doublet model = 2HDM with new “parity” ($\mathbb{Z}_2$-symmetry) [Deshpande, Ma, 1978; Barbieri et al, 2006, Lopez Honorez et al, 2006]:

- all known particles are $\mathbb{Z}_2$-even; second Higgs doublet $\phi_2$ is $\mathbb{Z}_2$-odd;
- $\mathbb{Z}_2$ parity is conserved $\rightarrow$ the lightest Higgs from $\phi_2$ is automatically stable $\rightarrow$ dark matter candidate!

Constraints from colliders, cosmology and DM searches $\rightarrow$ a lot of interest.

Dark matter can even help give tiny masses to neutrinos $\rightarrow$ scotogenic model [Ma, 2006].
Baryon asymmetry requires a strong first-order thermal electroweak phase transition in early Universe.

\[ \frac{v(T_c)}{T_c} \approx 1. \]

But it does not work in the SM!

We need to “additionally bend” the Higgs potential!
Cosmological phase transition

- Extra Higgses can produce **strong phase transition**!

- **2HDMs**: from early works [Turok, Zadrozny, 1992] to the recent detailed studies [Basler, Mühlleitner, Wittbrodt, 2018].

- Several minima → a **sequence of strong phase transitions**.

- Can be probed at future **GW observatories** [Caprini et al, 1512.06239].
Symmetries in 3HDM
What’s new in 3HDM compared to 2HDM:

- richer pheno (both scalar and fermion sectors);
- sophisticated potential → many minima, many options for phase transitions;
- combining nice features of 2HDM, e.g. NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009];
- new options for $CP$ violation, e.g. geometrical CPV [Branco, Gerard, Grimus, 1984], and for $CP$ symmetry, such as $CP4$ [Ivanov, Silva, 2015];
- symmetries, lots of symmetries in the 3HDM scalar sector!

  - General 3HDM potential

$$V = Y_{ab}(\phi_a^\dagger\phi_b) + Z_{ab,cd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d),$$

  contains 54 coefficients → challenging and impractical!
- **Symmetries** bring order and lead to robust features.
In early works: the choice of $G$ was driven mostly by guesses.

Full classification of symmetries possible in the 2HDM scalar sector was given in [Ivanov, hep-ph/0609018; Pilaftsis, 1109.3787].

**Within 3HDM scalar sector**, the full classification appeared recently:

- **abelian groups** [Ivanov, Keus, Vdovin, 1112.1660]
  
  $\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2, U(1), U(1) \times \mathbb{Z}_2, U(1) \times U(1).$

- **discrete non-abelian groups** [Ivanov, Vdovin, 1210.6553]
  
  $S_3, D_4, A_4, S_4, \Delta(54), \Sigma(36).$

- **symmetry breaking patterns** $G \rightarrow G_v$ [Ivanov, Nishi, 1410.6139]

- **interplay between $G$ and $CP$** [many classical works].
The original idea from 1970’s:
- extent $G$ to fermion sector,
- arrange for spontaneous violation $G \rightarrow G_v$,
- derive masses/mixing/CPV;

Many combinations of $G + \text{irreps} + \text{vevs}$ were tested, but
- if $G$ is large $\rightarrow$ severe problems in the quark sector;
  $A_4/S_4$ illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
- if $G$ is small $\rightarrow$ too many free parameters, no predictive power.

The obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]
- If the (active) Higgs sector is equipped with $G$, vevs must break $G$ completely to avoid unphysical $m_q$ or CKM.
- But for large $G$, this is algebraically impossible.
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Symmetry breaking in 3HDM

Strongest and weakest breaking of discrete symmetries in 3HDM and spontaneous CPV [Ivanov, Nishi, 1410.6139].

| group                  | $|G|$ | $|G_v|_{min}$ | $|G_v|_{max}$ | sCPv possible? |
|------------------------|------|--------------|--------------|----------------|
| abelian                | 2, 3, 4, 8 | 1 | $|G|$ | yes |
| $\mathbb{Z}_3 \times \mathbb{Z}_2^*$ | 6 | 1 | 6 | yes |
| $S_3$                   | 6 | 1 | 6 | — |
| $\mathbb{Z}_4 \times \mathbb{Z}_2^*$ | 8 | 2 | 8 | no |
| $S_3 \times \mathbb{Z}_2^*$ | 12 | 2 | 12 | yes |
| $D_4 \times \mathbb{Z}_2^*$ | 16 | 2 | 16 | no |
| $A_4 \times \mathbb{Z}_2^*$ | 24 | 4 | 8 | no |
| $S_4 \times \mathbb{Z}_2^*$ | 48 | 6 | 16 | no |
| $\Delta(27)$ CP-violating | 18 | 6 | 6 | — |
| $\Delta(27)$ CP-conserving | 36 | 6 | 12 | yes |
| $\Sigma(36)$            | 72 | 12 | 12 | no |
CP4 3HDM
Freedom of defining CP

In QFT, CP is not uniquely defined \textit{a priori}.

- phase factors $\phi(\vec{r}, t) \xrightarrow{CP} e^{i\alpha} \phi^*(-\vec{r}, t)$ [Feinberg, Weinberg, 1959],
- with $N$ scalar fields $\phi_i$, the general $CP$ transformation is

$$J : \phi_i \xrightarrow{CP} X_{ij} \phi_j^*, \quad X \in U(N).$$

If $\mathcal{L}$ is invariant under such $J$ with whatever fancy $X$, it is explicitly $CP$-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

- **NB**: The “standard” convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent!
Freedom of defining CP

\[ J : \phi_i \overset{CP}{\rightarrow} X_{ij}\phi_j^* , \quad X \in U(N), \]

Applying \( J \) twice leads to family transformation \( J^2 = XX^* \) which may be non-trivial. It may happen than only \( J^k = I \) (\( k = \) power of 2).

**CP-symmetry does not have to be of order 2**

The usual CP = CP2, the first non-trivial is CP4, then CP8, CP16, etc.

Models with higher-order GCP were known in 2HDM [Ferreira, Haber, Maniatis, Nachtmann, Silva, 2011] but they always led to accidental symmetries including the usual CP.
The question

What is the minimal multi-Higgs-doublet model realizing $\text{CP4}$ without accidental symmetries?
CP4 3HDM

The answer was given in [Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016].

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1\dagger 1) - m_{22}^2(2\dagger 2 + 3\dagger 3) + \lambda_1(1\dagger 1)^2 + \lambda_2 \left[(2\dagger 2)^2 + (3\dagger 3)^2\right] + \lambda_3(1\dagger 1)(2\dagger 2 + 3\dagger 3) + \lambda'_3(2\dagger 2)(3\dagger 3) + \lambda_4 \left[(1\dagger 2)(2\dagger 1) + (1\dagger 3)(3\dagger 1)\right] + \lambda'_4(2\dagger 3)(3\dagger 2),$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2} \left[(2\dagger 1)^2 - (3\dagger 1)^2\right] + \lambda_8(2\dagger 3)^2 + \lambda_9(2\dagger 3) \left[(2\dagger 2) - (3\dagger 3)\right] + h.c.$$ 

with real $\lambda_6$ and complex $\lambda_{8,9}$. It is invariant under $\text{CP4}$ $J : \phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad J^2 = \text{diag}(1, -1, -1), \quad J^4 = I.$$
Different versions of CP4 3HDM

- **DM CP4 3HDM:** unbroken CP4 with scalar DM candidates, similar to the inert doublet model in 2HDM. We assume that $\phi_2, \phi_3$ don’t get vevs $\rightarrow$ scalar DM candidates (stabilized by CP4!) with peculiar properties [Ivanov, Silva, 2016; Ivanov, Laletin, 2018].

  Scotogenic model for **radiative neutrino masses** based on CP4 rather than $\mathbb{Z}_2$ [Ivanov, 2018].

- **flavored CP4 3HDM:** CP4 is extended to the Yukawa sector and must be spontaneously broken $\rightarrow$ patterns in the flavor sector. [Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017]
DM CP4 3HDM

CP4-conserving minimum: \( v_i = (v, 0, 0) \). Expand the doublets as

\[
\phi_1 = \left( \frac{1}{\sqrt{2}} (v + h_{SM} + iG^0) \right), \quad \phi_2 = \left( \frac{1}{\sqrt{2}} (H^+ + ia) \right), \quad \phi_3 = \left( \frac{1}{\sqrt{2}} (h^+ + iA) \right).
\]

These fields are mass eigenstates; \( h \) and \( a \) are the DM candidates.

But they are not CP-eigenstates:

\[
H \xrightarrow{CP} A, \quad A \xrightarrow{CP} -H, \quad h \xrightarrow{CP} -a, \quad a \xrightarrow{CP} h.
\]

They can be combined into neutral complex CP-eigenstate fields

\[
\Phi = \frac{1}{\sqrt{2}} (H - iA), \quad \varphi = \frac{1}{\sqrt{2}} (h + ia), \quad \Phi \xrightarrow{CP} i\Phi, \quad \varphi \xrightarrow{CP} i\varphi.
\]

Conserved quantum number: not CP-parity but CP-charge \( q \) defined mod 4.
Suppose due to some non-thermal process the starting values \( n(\varphi) \neq n(\varphi^*) \rightarrow \) asymmetric DM. Will this asymmetry survive?

- Unlike in typical asymmetric DM models, \( n(\varphi) - n(\varphi^*) \) is not fixed.
- In the \( \varphi \)-dominated situation, there is no direct annihilation: \( \varphi \varphi \not\rightarrow \text{SM} \).
- But there exists regeneration \( \varphi \varphi \rightarrow \varphi^* \varphi^* \), so that \( \varphi \varphi^* \rightarrow \text{SM} \) is now possible → two-stage annihilation.
- We explored the competition between the two processes in [Ivanov, Laletin, 1812.05525].
Asymmetric DM regime

The competition between the annihilation $\varphi \varphi \not\to$ SM and conversion $\varphi \varphi \leftrightarrow \varphi^* \varphi^*$ due to

$$\frac{\lambda_{\text{conv}}}{4!} [\varphi^4 + (\varphi^*)^4]$$

affects the thermal evolution of the asymmetry:

$$\delta = \frac{n_\varphi - n_{\varphi^*}}{n_\varphi + n_{\varphi^*}}.$$ 

Whether the asymmetry survives or not sharply depends on $\lambda_{\text{conv}}!$

If evolution starts at $x = 1$, the boundary is $\lambda_{\text{conv}} \sim 10^{-5}$. 
Extending CP4 to the Yukawa sector: $\psi_i \rightarrow Y_{ij}\psi_j^{CP}$, where $\psi^{CP} = \gamma^0 C\bar{\psi}^T$.

$$\bar{Q}_L \Gamma_a d_R \phi_a + \bar{Q}_L \Delta_a u_R \bar{\phi}_a + h.c.$$ is invariant under CP4 with known $X_{ab}$ if

$$(Y^L)\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^* , \quad (Y^L)\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^*.$$ We solved these equations: cases $A, B_1, B_2, B_3$. 
CP4-symmetric quark sector

**case B₁**

\[ \Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}. \]

**case B₂**

\[ \Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}. \]

**case B₃**

\[ \Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}. \]
Numerical scan

1. Scalar sector scan:
   - stick to the scalar alignment, take $h_{125}$ to be the lightest scalar, vary 9 free parameters: $v_3/v_2$, $u/v_1$, and 7 $\lambda$'s;
   - simplified checks of boundedness from below and perturbativity;
   - check that $S$, $T$, $U$ parameters are within 3$\sigma$ of expt.

2. Yukawa sector scan
   - fit all quark masses, mixing, and CPV phase (easy);
   - add $K$ and $B$ oscillation parameters $|\epsilon_K|$, $\Delta m_K$, $\Delta m_{B_d}$, $\Delta m_{B_s}$ via expressions from [Buras et al, 2013] (tree-level contributions from neutral Higgses only).

3. Results: many good points found, but they lead to rather light $H^{\pm} \rightarrow$ probably ruled out by the very recent $t \rightarrow H^{+}q'$ and $H^{+} \rightarrow q\bar{q}'$ LHC results.
   - But the scan can be repeated by relaxing certain assumptions $\rightarrow$ good parameter space regions are to be expected.
Conclusions

- 3HDMs offer a **richer list of opportunities** than 2HDM: flavour, CPV, scalar pheno, astroparticle, cosmology.

- (Approximate) symmetries and their breaking play crucial role.

- Classification of symmetry-related situation and basis-invariant methods now exist → it’s time to explore 3HDM phenomenology systematically.

- **CP4 3HDM** is a **hidden gem of 3HDM**: the simplest model based on a higher-order $CP$ symmetry, with remarkable structural properties and unusual phenomenology. It is worth exploring in detail.